Math 36500: Combinatorics: Final Review
Prof. Hooper

Disclaimer. This test is just a recommendation of things to study and problems to work on. You may be asked about things that do not appear here. You should practice doing problems from the book in addition to the problems included in this sheet.

Test makeup. The test will include 6-8 problems, some with multiple parts. The format will be similar to the midterms. You will not need to simplify your answers, so you do not need a calculator or other electronic device. Only simple non-graphing calculators can be used. (Nothing that can store information.)

With the exam, I will distribute the tables of information about generating functions that I distributed in class and posted on the website.

Covered Material.

- Approximately half the exam will cover material from Chapters 4 and 5. Specifically, §4.1-4.5 and §5.1.
- Approximately a quarter of the exam will cover material from the first midterm: §0.1-0.4 and §1.1-1.5.
- Approximately a quarter of the exam will cover material from the second midterm: §2.1-2.5, the Pigeonhole Principle (which appears in §3.2), and §1.6.

This is the goal, the actual proportion of problems devoted to each section may be somewhat different (and is subjective especially since some questions may involve ideas from multiple sections).

Definitions: You need to be able to understand and use the following definitions:

disjoint, word, subset, power set, Cartesian Product, union, intersection, difference (for sets), cardinality, Venn diagram, function, domain, codomain, range, closed formula, recursive definition, initial conditions, recurrence relation, **surjection (onto)**, **injection (one-to-one)**, *bijection, image, inverse image, bit string, lattice path, binomial coefficients, Pascal's triangle, permutation, k-permutation of n elements, binomial identity, combinatorial proof, stars and bars chart, derangement, sequence, Fibonacci numbers, sequence of partial sums,* **arithmetic sequence**, **geometric sequence**, *triangular numbers, difference sequence, Δ^k-constant, solving a recurrence relation, characteristic equation, characteristic roots, characteristic polynomial, base case, inductive case (or inductive step),* **graph**, *vertex, edge, adjacent, isomorphic, isomorphism,* **subgraph**, **induced subgraph**, *connected graph*, degree of a vertex, degree sequence,* **bipartite graph**, *complete graph, complete bipartite graph, path, cycle, tree, forest, leaf of a tree, spanning tree, planar graph, planar representation, face, polyhedron, convex polyhedron,* **vertex coloring**, **proper vertex coloring**, **chromatic number**, *clique, clique number, edge coloring, proper edge coloring, chromatic index,* *walk, Euler path, Euler circuit, Hamilton path, Hamilton cycle, generating series, generating function.*

You should be able to define the terms in bold above. (Memorize the definitions!)

The book has lists of definitions in §0.4: Functions and §4.1: Graphs.

Main results: You should understand how to use the following results:

Additive Principle, Multiplicative Principle, Formula for the number of k-permutation of n elements, Closed formula for $\binom{n}{k}$, Principle of Inclusion/Exclusion (PIE) (for 2 sets, 3 sets and "advanced"), Finite Differences (a sequence is polynomial if and only if it is Δ^k-constant for some k), Principle of Mathematical Induction (§2.5), The Pigeonhole Principle, The Handshake Lemma, Paths in Trees (Proposition 4.2.1), Trees have leaves (Proposition 4.2.3), Number of edges and vertices in a tree (Proposition 4.2.4), Connected graphs have spanning trees, Euler's Formula for Planar Graphs, K_5 and $K_{3,3}$ are not planar, The four color theorem, When graphs have Euler Paths and Circuits (§4.5).

Problem types: You should be able to do the following:

- Count sets and functions, using the Additive and Multiplicative Principles, Stars and Bars, the Principle of Inclusion/Exclusion (PI/E) (e.g., §1.7 # 1, 3, 5, 9, 11, 12)
- Use the binomial coefficients to solve problems (e.g., §1.7 # 7, 8; §2.6 # 9)
- Give a combinatorial proof (e.g., §1.5 # 2, 3, 6, 14; §1.7 # 15)
- Understand problems making use of sets, functions, and related terminology (e.g., §1.1 # 5, 11; §1.7 # 17, 18)
- Compute partial sums of arithmetic and geometric sequences (e.g., §2.6 # 1-3)
- Work out closed formulas for polynomial sequences (e.g., §2.6 # 5, 6, 7)
- Work out closed formulas for recursive sequences (e.g., §2.6 # 10, 11, 12)
- Give inductive proofs of statements and formulas, especially on topics related to the course (e.g., §2.6 # 14, 15, 16, 17)
- Make use of the Pigeonhole principle to solve problems (e.g., §3.2 # 18, 19).
- Tell if two graphs are isomorphic (e.g., §4.1 # 3, 4, 5; §4.7 # 1, 5ab)
- Make use of Euler's Formula for Planar Graphs, including using it to prove a graph is not planar (e.g., Theorems 4.3.1 and 4.3.2; §4.3 # 11; §4.7 # 6, 7c, 10, 16, 19)
- Prove statements about graphs using induction (e.g., §4.3 # 8, 9; §4.4 # 6, 11; §4.7 # 23)

- Combine and synthesize ideas and techniques in the course to solve novel problems, especially involving graphs (e.g., §4.7 # 7, 8, 9, 12, 14)
- Convert between sequences and generating functions and understand how manipulations of each relate (e.g., §5.1 # 1, 2, 5, 8, 14, 16)

The list above is not comprehensive. You should be able to do more with what you learned than simply what is stated above, which is overly simplistic. For instance, using knowledge obtained from polynomial fitting (which works out a closed formula for a polynomial sequence), you should be able to prove a sequence is not a polynomial sequence. In particular, questions will be asked that require you to combine multiple techniques, or to use ideas non-standard ways to solve problems. (Though, if you can do all that is listed above, you'll presumably do fairly well on the final.)