

Math 36500: Combinatorics: Practice Final 1

For each $n \geq 1$, let c_n be the number of ways the number n can be written as a sum of numbers from the set $\{1, 2, \dots, n\}$ where the sum can include just a single number, and sums written in a different order are considered different.

The chart below shows the first few values of c_n and the sums.

n	c_n	Sums adding to n
$n = 1$	$c_1 = 1$	1
$n = 2$	$c_2 = 2$	2, 1 + 1
$n = 3$	$c_3 = 4$	3, 2 + 1, 1 + 2, 1 + 1 + 1.

- (a) Explain why for every $n \geq 2$, we have $c_n = 1 + \sum_{k=1}^{n-1} c_k$.
- (b) Use strong induction to prove that because $c_1 = 1$ and $c_n = 1 + \sum_{k=1}^{n-1} c_k$ for every integer $n \geq 2$, we must have that $c_n = 2^{n-1}$ for every n . (This does not require you to complete (a) successfully!)