CONTINUITY OF ABSOLUTE VALUE

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Exercise. Use the ϵ-δ definition of continuity to prove that $f(x) = |x|$ is continuous.

Proof. We will show $f(x) = |x|$ is continuous at x_0. Let $\epsilon > 0$. We must find a $\delta > 0$ so that $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \epsilon$.

Let $\delta = \epsilon$ and choose an x so that $|x - x_0| < \delta$. We break into two cases.

- If both x and x_0 are non-negative or both are non-positive then
$$\big||x| - |x_0|\big| = |x - x_0| < \delta = \epsilon.$$
- If x and x_0 have opposite signs then $|x - x_0| = |x| + |x_0|$. By the triangle inequality,
$$\big||x| - |x_0|\big| \leq \big||x|\big| + \big| - |x_0|\big| = |x| + |x_0| = |x - x_0| < \delta = \epsilon.$$

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