Math 346: Linear Algebra: Take Home Quiz 2

Name: _____

Instructions: Solve these problems without discussing with other people. Cite any sources you use other than the textbook. **There are two problems. One is on the back!**

1. (10 points) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and

$$T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}.$$

   (a) What are $T\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $T\begin{pmatrix} 0 \\ 1 \end{pmatrix}$?

   (b) Find a matrix A so that $T(\mathbf{v}) = A\mathbf{v}$ for all $\mathbf{v} \in \mathbb{R}^2$.

2. (10 points) (a) Suppose C is an invertible matrix and $AC = BC$. Prove that $A = B$.

   (b) Let $C = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ which is not invertible. Find two 2×2 matrices A and B so that $A \neq B$ but $AC = BC$.