# Math 346: Linear Algebra: Take Home Quiz 1

**Solutions:**

1. (5 points) Find a $3 \times 3$ matrix $A$ so that

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y+z \\ x \\ y \end{pmatrix} \quad \text{for all real numbers } x,\ y \text{ and } z.$$

**Solution:** The columns of the matrix $A$ are given by

$$A \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \text{and} \quad A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

Thus $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

2. Consider the linear system
$$\begin{aligned} 2x + y &= 0 \\ -2x - 3z &= 1 \\ 6x + y + 8z &= 2. \end{aligned}$$

   (a) (5 points) What is the augmented matrix of the system?

**Solution:**

$$\left(\begin{array}{rrr|r} 2 & 1 & 0 & 0 \\ -2 & 0 & -3 & 1 \\ 6 & 1 & 8 & 2 \end{array}\right).$$

   (b) (5 points) Apply elimination to the augmented matrix until it has the form $[U : \mathbf{c}]$ where $U$ is an upper triangular matrix.

**Solution:**

$$\left(\begin{array}{rrr|r} 2 & 1 & 0 & 0 \\ -2 & 0 & -3 & 1 \\ 6 & 1 & 8 & 2 \end{array}\right) \sim_1 \left(\begin{array}{rrr|r} 2 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & -2 & 8 & 2 \end{array}\right) \sim_2 \left(\begin{array}{rrr|r} 2 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 2 & 4 \end{array}\right).$$

In step $\sim_1$ we add row 1 to row 2 and subtract 3 times row 1 from row 3. In step $\sim_2$ we add twice row 2 to row 3.

(c) (5 points) Apply back substitution to solve the system. (Check your answer against the system!)

**Solution:** The final matrix above corresponds to the system of equations:

$$\begin{aligned} 2x + y &= 0 \\ y - 3z &= 1 \\ 2z &= 4 \end{aligned}$$

Solving $2z = 4$ for $z$ yields $z = 2$. Plugging this value into the second equation yields $y - 6 = 1$ so $y = 7$. Plugging the value of $y$ into the first equation yields $2x + 7 = 0$ so $x = \frac{-7}{2}$. This gives a unique solution: $x = \frac{-7}{2}$, $y = 7$ and $z = 2$.