Syllabus for Math 70100, Fall 2014 Semester

Prof. Hooper

This course is the first of two classes which are intended to prepare students for passing the qualifying exam in analysis. I expect to cover all the material for this course listed in the topics for the qualifying exam here:

`http://www.gc.cuny.edu/CUNY_GC/media/CUNY-Graduate-Center/PDF/Mathematics/Real-Syllabus.pdf?ext=.pdf`

We will certainly cover a few things not listed there.

Prerequisite Material

Some material listed in the document above should have been seen by any undergraduate mathematics major. However, you are encouraged to review this material:

- The real numbers, including the least upper bound property.
- Elementary aspects of cardinal numbers (countable and uncountable sets, the uncountability of the reals).
- Differentiation in several variables, including the Inverse and Implicit Functions Theorems.

Please let me know if you have not seen these topics in prior coursework.

Tentative Course Syllabus

Topological Spaces:

- Definition of topological space, normed vector space, and metric space, completeness.
- Examples: $\mathbb{R}^n$ and function spaces.
- Continuous maps between topological spaces.
- Uniform convergence.
- Connectedness properties.
- Compactness.
- The Heine-Borel Theorem.

- Sequential compactness and the Bolzano-Weirstrass theorem.
- Example: The Cantor set.
- Separation by continuous functions: Urysohn's Lemma.
- The Baire Category Theorem and applications.

Continuous Functions on Compact Sets:

- The Stone-Weierstrass Theorem.
- Ascoli's Theorem.

Measures:

- σ-algebras.
- Definition of a measure, and basic properties.
- Outer Measures and Carathéodory's Theorem.
- Borel measures on $\mathbb{R}$ and Lebesgue measure.

Integration:

- Measurable functions.
- Integration.
- The Lebesgue integral and comparison to the Riemann integral.
- Convergence in measure, Egoroff's Theorem, and Lusin's Theorem.
- Product measures including Lebesgue measure on $\mathbb{R}^n$.

Signed measures and differentiation:
Given time, we will also cover the following:

- Signed measures.
- Absolute continuity of measures and mutually singular measures.
- The Radon-Nikodym derivative, and the Lebesgue-Radon-Nikodym theorem.
- Differentiation on $\mathbb{R}^n$.
- Functions of bounded variation on $\mathbb{R}$.