Math A4500: Final Exam Study Guide

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Disclaimer. This test is just a recommendation of things to study. You may be asked about things that do not appear here.

Test Layout. The final exam will be structured slightly differently than the prior midterms. First, it will test material not covered by the midterms.

- There will be a test problem covering §2.5: Attractors. Either the question will be about abstract properties of attractors, or will investigate simple examples.
- There will be a test problem asking about §2.6: Stable and unstable manifolds. For example, you may be asked to describe the unstable and stable manifolds of a simple map. Or, you may be asked to prove that a fixed or periodic point is hyperbolic and to classify it.

Second, there will be a list of approximately 5 problems and you will need to several (maybe 3) to solve. These problems will be used to test your understanding of the many examples we have covered in detail in the course. It will also test your knowledge of ideas and terms from the theory of dynamical systems. Systems that may appear, with example topics of interest in parenthesis:

1. The Logistic Family. (How does the Schwarzian derivative restrict the dynamics of a map in the Logistic Family? What bifurcations occur for small parameters? Which small parameters are structurally stable? What happens for parameters $\mu > 4$?)
2. 1- and 2-sided shift spaces, especially full shift spaces. (Analyze the dynamics of the shift map. What are the periodic points? Are they dense? Is there a dense orbit? Is the shift map chaotic?)
3. The dynamics of linear maps in $\mathbb{R}^2$ and $\mathbb{R}^3$. (Fix a linear map. Is it hyperbolic? If so, what are the stable and unstable spaces? Describe the orbits of points.)
4. The Smale Horseshoe of §2.3. (Is there an attractor? Which points are not forward asymptotic to the attracting fixed point? Which points have infinite backward orbits? Code the dynamics of the points Λ which are not forward asymptotic to the fixed point and have infinite backward orbits. Use the coding scheme to prove that the dynamics on this set is chaotic.)
5. Toral Automorphisms of §2.4. (Fix a hyperbolic toral automorphism. What can be said about the periodic points. What are the stable and unstable manifolds of a periodic point? Find homoclinic and heteroclinic points. Does the automorphism have sensitive dependence on initial conditions? Use a Markov Partition to code the dynamics.)
6. The solenoid of §2.5. (Is the solenoid an attractor? Show that the dynamics on the solenoid are topologically conjugate to an inverse limit space. Use the inverse limit construction to answer dynamical questions about the map. Is the map chaotic? What are the stable and unstable sets?)

The above are just sample questions. You should know the above examples well, and be able to improvise (i.e. answer questions not appearing above).

Sections that may be covered. §1.1-1.12 and §2.1-2.6.

List of Definitions. You may be asked to define and/or use several terms on the test. The following is a list of important terms in the book.

class C^r, one-to one, onto, homeomorphism, C^r-diffeomorphism, limit point, closed set, open set, dense, forward orbit, backward orbit, orbit, periodic point, period, prime period (or least period), eventually periodic, forward and backward asymptotic, critical point, hyperbolic periodic point, multiplier, attracting periodic point, repelling periodic point, Cantor set, totally disconnected, perfect, sequence space or (shift space), shift map, itinerary, topological conjugacy, topologically transitive, sensitive dependence on initial conditions, chaotic, topologically semi-conjugate, C^r-distance, C^r-structurally stable, Schwarzian derivative, first return map, diffeomorphism (of $\mathbb{R}^2$), stable and unstable sets of a point, forward and backward asymptotic, homoclinic, heteroclinic, Markov partition, trapping region, attractor, inverse limit space, stable and unstable manifold

You are expected to know the definitions given in the book. (I will not ask you to to define terms for which no formal definition is given in the book.)