Math 290-2: Practice Problems for Midterm II

1. Definitions.

   (a) "Let $\mathbf{f} : X \subseteq \mathbb{R}^n \to \mathbb{R}^m$ and let $\mathbf{a} \in X$. Then $\mathbf{f}$ is **continuous** at $\mathbf{a}$ if ..."

   (b) Let $X \subseteq \mathbb{R}^n$. A **neighborhood** of a point $\mathbf{x} \in X$ is

   (c) "The **graph** of a function $\mathbf{f} : X \subseteq \mathbb{R}^n \to \mathbb{R}^m$ is ..."

2. True/False.

   (a) The two sets of parametric equations $x = 4-t$, $y = -1+3t$, $z = 2t$ and $x = 3+2t$, $y = 2-6t$, $z = 1-4t$ both represent the same line.

   (b) For any vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ in $\mathbb{R}^3$, we have $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$.

   (c) If $f : \mathbb{R}^2 \to \mathbb{R}$ and both $f_x(a,b)$ and $f_y(a,b)$ exist, then f is continuous at (a,b).

   (d) The range of $f(x,y) = (x, x^2+y^2, e^y)$ is $\{(x,y,z) \in \mathbb{R}^3 \mid y \geq 0 \text{ and } z > 0\}$.

   (e) A differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ has continuous partial derivatives.

   (f) The range of the function $f(x,y) = (\cos(x^2+y), \sin(x^2+y))$ is $\mathbb{R}^2$.

   (g) The solution set to the equation $17x^2 - 5y^2 - 2z^2 = 53$ is a hyperboloid of one sheet.

   (h) The plane $3x + 2y - 4z = 2$ and the line $\mathbf{r}(t) = (1, 6t+2, 3t-2)$ intersect.

   (i) The function $f(x,y) = (x-y)^{\frac{1}{3}}$ is differentiable at the point $(3,3)$.

   (j) Parametric equations for the plane passing through $(1,1,2)$ with normal vector $\begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix}$ are $\begin{cases} x &= 5 + 2s - 3t \\ y &= s \\ z &= t \end{cases}$

   (k) The graph of every scalar-valued function of two variables is a level set of a scalar-valued function of three variables.

3. Evaluate the following limits if they exist. Demonstrate the correctness of you answer.

   (a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy^{\frac{4}{3}}}{x^2+y^2}$

   (b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{(x+y)^4}{x^4+y^4}$

   (c) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2 - 3xy + 5y^3 - 7}{x+y+1}$

4. Consider the solid whose cylindrical coordinates satisfy $r \leq z \leq \sqrt{2-r^2}$. *Tip: Do the three parts of this question in whichever order makes the most sense to you.*

   (a) Describe the solid using Cartesian coordinates.

   (b) Sketch the solid.

(c) Describe the solid using spherical coordinates. (There is a very simple description. To receive full credit, this is the one you should give.)

5. Consider the surface S described by the following equation.

$$x^2 + 4y^2 - z^2 = 0$$

   (a) Find a formula for the curve obtained by intersecting S with the plane $z = 16$. Describe this intersection in words.
   (b) Find a formula for the curve obtained by intersecting S with the plane $y = 1$. Describe this intersection in words.
   (c) Graph the surface S.
   (d) What type of surface is S?

6. Find the distance between the point $(3, -2, 7)$ and the plane $4x - 6y + z = 5$.

7. Let A be the point $(1, 2, -1)$, let B be the point $(2, 0, 1)$, and let C be the point $(-1, 1, 1)$.
   (a) Find the area of the triangle with vertices A, B, and C.
   (b) Find an equation for the plane containing the points A, B, and C.
   (c) Find a set of parametric equations for the line passing through point A that is perpendicular to the plane containing the points A, B, and C.

8. Consider the surface S in $\mathbb{R}^3$ described by the following equation.

$$x^2 - 2x + y^2 + \frac{z^2}{4} = 0$$

   (a) Sketch and describe the surface S.
   (b) Find a formula for S in cyllinderical coordinates.

9. Consider the function $f(x, y) = x^2 - 4y^2$.
   (a) Draw several level sets of this function in the xy-plane.
   (b) Use your work from the previous part to sketch the graph of f.
   (c) Name the surface.
   (d) Find an equation for the tangent plane to the graph $z = f(x, y)$, at the point $(3, 1, 5)$.

10. Prove that every linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable.