

Math290-2, Section 61
Example graph of surface

Example: Consider the function

$$F(x, y, z) = x^2 + y^2 - z^2.$$

Graph the level sets $F = 0$, $F = 1$, and $F = -1$.

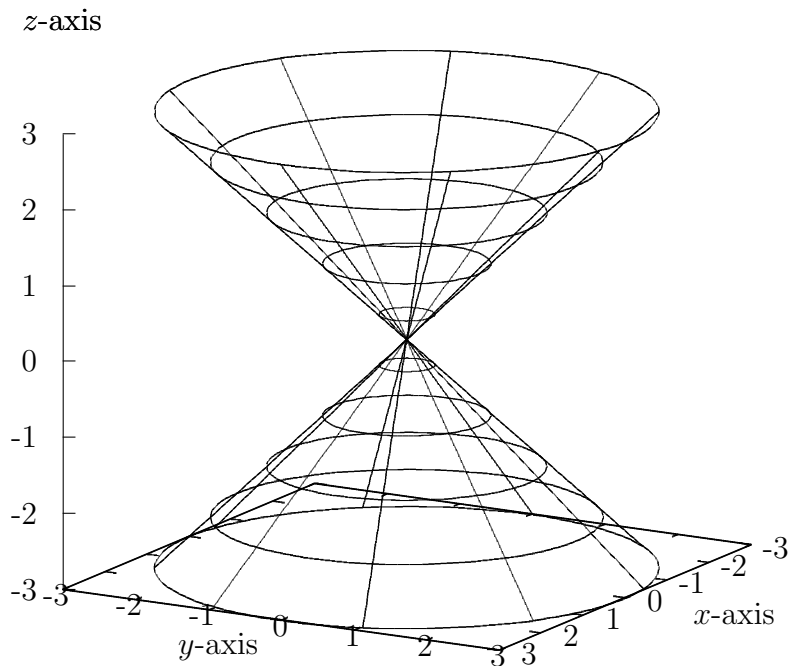
Answer: We start with $F = 0$. Simplifying we see

$$x^2 + y^2 = z^2.$$

The section of the graph by the plane $z = k$ is a circle of radius $|k|$.

Here is the graph. The surface is a **cone**.

The graph of $x^2 + y^2 - z^2 = 0$



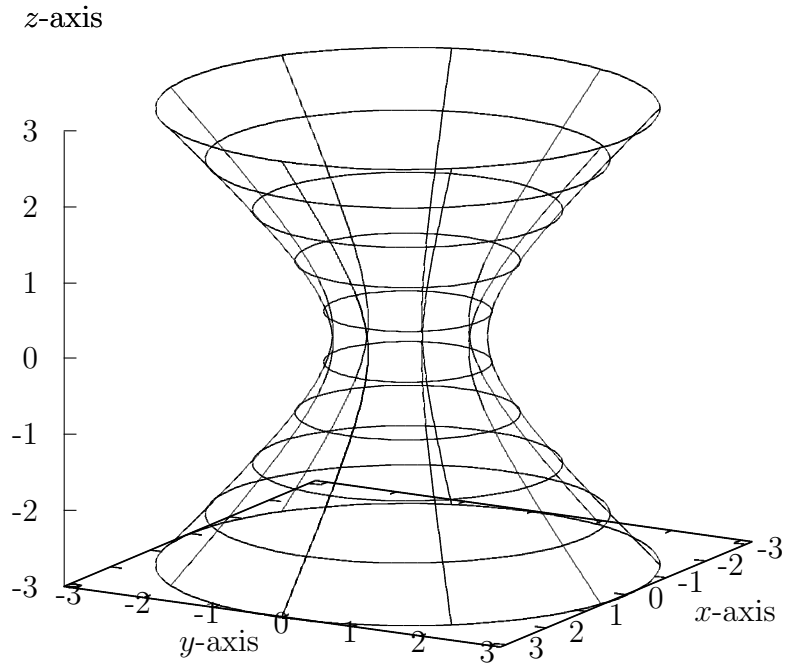
Now, we deal with the case $F(x, y, z) = x^2 + y^2 - z^2 = 1$. Then we have

$$x^2 + y^2 = z^2 + 1.$$

The section of the graph by the plane $z = k$ is a circle of radius $\sqrt{k^2 + 1}$.

Here is the graph. The surface is called a **hyperboloid of one sheet**.

The graph of $x^2 + y^2 - z^2 = 1$



Finally, we deal with the case $F(x, y, z) = x^2 + y^2 - z^2 = -1$. Then we have

$$x^2 + y^2 = z^2 - 1.$$

The section of the graph by the plane $z = k$ is a circle of radius $\sqrt{k^2 - 1}$. This radius is only defined when $|k| > 1$, so there is no solution when $-1 < z < 1$.

Here is the graph. The surface is called a **hyperboloid of two sheets**.

The graph of $x^2 + y^2 - z^2 = -1$

