## Math 290-3: Practice problems for the first midterm

### Remarks on the midterm

- Definitions will now be graded with only limited partial credit, so most definitions will receive either full or no credit. However, to make studying easier, we will provide a list of possible definitions that may appear on the midterm or exam. The following terms may appear on this midterm.
  - "Given a closed rectangle $R = [a, b] \times [c, d]$, a **partition of** $R$ **of order** $n$ is ..."
  - "Given a function $f$ from $R = [a, b] \times [c, d]$ to $\mathbb{R}$, a **Riemann sum** of $f$ on $R$ is ..."
  - "Let $R$ be a closed rectangle in $\mathbb{R}^2$. If $f : R \to \mathbb{R}$ is integrable on $R$, then the **double integral** of $f$ on $R$ is ..."
  - "Given a closed box $B = [a, b] \times [c, d] \times [p, q]$, a **partition of** $B$ **of order** $n$ is ..."
  - "Given a function $f$ from $B = [a, b] \times [c, d] \times [p, q]$ to $\mathbb{R}$, a **Riemann sum** of $f$ on $B$ is ..."
  - "Let $B$ be a closed box in $\mathbb{R}^3$. If $f : B \to \mathbb{R}$ is integrable on $B$, then the **triple integral** of $f$ on $B$ is ..."
  - "Let $\mathbf{T} : \mathbb{R}^n \to \mathbb{R}^n$ be differentiable. The **Jacobian** of $\mathbf{T}$ is ..."
- You will also be required to know the statements of a few main results. Statements of the following may be required for the midterm.
  - Proposition 1.1: "The volume under the graph of a continuous non-negative function $f : [a, b] \times [c, d] \to \mathbb{R}$ and over the rectangle $[a, b] \times [c, d]$ is ..."
  - Theorem 2.5: Sufficient conditions for integrability.
  - Theorem 2.6: Fubini's theorem for double integrals.
  - Theorem 5.3: The change of variables theorem in double integrals.

### Practice problems for the midterm

For additional practice problems, consider the exercises in §5.7-5.8 of the book.

1. Prove the following statement. If $f : [a, b] \to \mathbb{R}$ and $g : [c, d] \to \mathbb{R}$ are continuous, then
$$\int_a^b \int_c^d f(x)g(y)\,dydx = \left(\int_a^b f(x)\,dx\right)\left(\int_c^d g(y)\,dy\right)$$
2. Reverse the order of integration of $\displaystyle\int_0^1 \int_0^x \sqrt{x^3 + y^3}\,dydx$.
3. Is the following statement true or false? If $W \subseteq \mathbb{R}^3$ is compact and $\partial W$ has volume zero, then the volume of $W$ is
$$\iiint_W 1\,dV$$

4. Evaluate the following iterated integral.

$$\int_0^1 \int_y^1 \int_0^y 2\pi z \sin(\pi x^4)\ dz\ dx\ dy$$

5. Rewrite the following integral as an integral (or sum of integrals) in the order $\iiint f\ dx\ dy\ dz$.

$$\int_0^1 \int_0^1 \int_0^{xy} f\ dz\ dy\ dx$$

6. Consider the region $R$ which lies above the x-axis and below the first hump of the catenoid parameterized by the curve

$$\mathbf{r}(t) = (t - \sin t, 1 - \cos t) \quad \text{for } 0 \leq t \leq 2\pi.$$

This region can be parameterized by the function $\mathbf{T} : [0, 2\pi] \times [0, 1] \to R$ given by

$$\mathbf{T}(u, v) = \big(u - \sin u, v(1 - \cos u)\big).$$

Use this information to compute the area of $R$.

7. Express the triple integral

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} x^2 + y^2\, dz\, dy\, dx$$

as a triple integral in cylindrical coordinates. Do not evaluate the integral.

8. Consider the region $R$ between the planes $z = 1$ and $z = -1$ and inside the hyperboloid

$$x^2 + y^2 = z^2 + 1.$$

Evaluate the integral

$$\iiint_R x^2 + y^2 + z^2\ dV.$$

9. Find the volume inside the sphere $x^2 + y^2 + (z - 1)^2 = 1$ but outside the sphere $x^2 + y^2 + z^2 = 2$. (*Hint:* Carefully looking at the geometry of the region may make your job easier.)