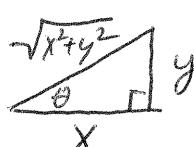


Math 230-0, Section 57: Quiz ~~X~~ 3  
 October, 12 2006

1. (5 points) The polar equation

$$r^2 = \frac{4}{3 \sin^2(\theta) + 1} \quad (1)$$

determines a curve in the plane. Find a simple formula for this curve in Cartesian (rectangular) coordinates. What is the name of this curve?

Recall  $r = \sqrt{x^2 + y^2}$      $\theta = \tan^{-1}\left(\frac{y}{x}\right)$     

To compute  $\sin\theta$ , consider the triangle

Thus  $\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$ . Thus (1) is equivalent to

$$x^2 + y^2 = \frac{4}{3 \frac{y^2}{x^2 + y^2} + 1}$$

$$3y^2 + x^2 + y^2 = 4 \text{ or equivalently } \boxed{x^2 + 4y^2 = 4}.$$

An ellipse!

2. (5 points) Recall the definitions of hyperbolic sine and cosine:

$$\cosh(t) = \frac{e^t + e^{-t}}{2} \quad \sinh(t) = \frac{e^t - e^{-t}}{2}$$

Use cosh and sinh to parameterize the hyperbola determined by the equation

$$x^2 - 6x = y^2$$

(Hint: Complete the square.)

$$x^2 - 6x + 9 = y^2 + 9$$

$$(x-3)^2 - y^2 = 9$$

$$\frac{(x-3)^2}{9} - \frac{y^2}{9} = 1$$

$$x(t) = \pm 3 \cosh t + 3$$

$$y(t) = 3 \sinh t$$

3. (5 points) Consider the vector-valued function

$$\mathbf{r}(t) = 3e^t \sin(t)\mathbf{i} + 3e^t \cos(t)\mathbf{j} + 4e^t\mathbf{k}$$

Let  $L_{[a,b]}$  denote the length of the arc traced out by  $\mathbf{r}(t)$ , for  $a \leq t \leq b$ .

- (a) (5 points) Find a Cartesian equation for a quadric surface containing this curve. Name the surface. Note:  $\sin^2 + \cos^2 = 1$

$$x^2 + y^2 = 9e^{2t} \quad \text{and} \quad z^2 = 16e^{2t}$$

- (b) (5 points) Find  $L_{[0,1]}$ .

$$\text{so } \boxed{\frac{16}{9}(x^2 + y^2) = z^2} \quad \text{A cone!}$$

$$\mathbf{r}'(t) = \langle 3e^t \cos t + 3e^t \sin t, 3e^t \cos t - 3e^t \sin t, 4e^t \rangle$$

$$\|\mathbf{r}'(t)\| = \sqrt{9e^{2t}((\cos t + \sin t)^2 + (\cos t - \sin t)^2) + 16e^{2t}} = \sqrt{34}e^t$$

$$L_{[0,1]} = \int_0^1 \sqrt{34} e^t dt = \left[ \sqrt{34} e^t \right]_0^1 = \boxed{\sqrt{34}(e-1)}$$

- (c) (5 points) We define the arc length of the arc with  $t \leq b$  to be

$$L_{(-\infty, b]} = \lim_{a \rightarrow -\infty} L_{[a, b]}$$

Find  $L_{(-\infty, 0]}$ . This is the length traced out by  $\mathbf{r}(t)$  for  $t \leq 0$ .

$$L_{[a, 0]} = \int_a^0 \sqrt{34} e^t dt = \sqrt{34} \left( \cancel{0} - e^a \right)$$

$$\lim_{a \rightarrow -\infty} \sqrt{34} (1 - e^a) = \boxed{-\sqrt{34}}$$