List of my results

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1 Veech Triangle

The $(\pi/12, \pi/3, 7\pi/12)$ triangle is Veech. I can explicitly compute generators for the Veech group. *This is written up on my web page.*

2 Bounding box results

1. Right triangles have no stable periodic billiard paths. *This is written up on my web page, and submitted to a journal.*

2. Fixing the sequence of edges of a triangle a billiard path must hit, I can compute a bounding box for the set of all triangles that realize that sequence by periodic billiard path. (This has generalizations to other polygons, and to the search for geodesics in Euclidean cone manifolds.) *This argument is written up, but not the generalizations. It is posted as a work in progress on my website.*

3. Item 2 implies that the sequence of edges arising from a stable periodic billiard path in an acute triangle can not also arise from an obtuse triangle. The statement is proved by showing that the bounding box I compute is connected and avoids the set of right triangles. *Unwritten.*

3 Stability and Isosceles triangles

1. Non-Veech obtuse isosceles triangles admit stable periodic billiard paths. This is proved associating to each set of triangles

$$I_n = \{\text{triangles } (x, x, \pi - 2x) | \frac{\pi}{2n+2} < x < \frac{\pi}{2n}\} \text{ where } n \in \mathbb{N} \text{ and } n > 2 \tag{1}$$

   to a family of periodic billiard paths which I prove cover the interval explicitly. *This is as yet unwritten.*

2. Veech triangles of the form $(\pi/2^n, \pi/2^n, *)$ do not admit stable periodic billiard paths. There is a topological double covering from the translation surface associated to $(\pi/2^{n+1}, \pi/2^{n+1}, *)$ to the the translation surface associated to $(\pi/2^n, \pi/2^n, *)$. This allows you to inductively show that geodesics on the cone manifold obtained by doubling each such triangle along its edges (obtaining a sphere with 3 cone points) are never null homologous. By a well known result (A billiard path is stable iff null homologous on this cone manifold), these triangles admit no stable periodic billiard paths. *Unwritten*

Since you may be interested, I'll tell you where these Isosceles triangle results are going. Rich Schwartz and I have a list of conjectured stable periodic billiard paths in the remaining obtuse isosceles Veech triangles. This is probably easily provable, but we have not gotten to it. Rich and I expect these $(\pi/2^n, \pi/2^n, *)$ Veech triangles to satisfy the same bad properties as the 30-60-90 triangle. That is, we also believe that you need infinitely tiles (sets of triangles with billiard paths of some fixed combinatorial type) to cover a neighborhood of the $(\pi/2^n, \pi/2^n, *)$ triangles. I believe I am close to a proof of this. Nevertheless, we expect that you can cover a neighborhood of each of these triangles. This is probably difficult to prove.